

2018

AP[®]

 **CollegeBoard**

AP Physics C: Electricity and Magnetism

Free-Response Questions

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^8$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23} \text{ mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11} (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31 \text{ J}/(\text{mol}\cdot\text{K})$	Acceleration due to gravity at Earth's surface, $g = 9.8 \text{ m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23} \text{ J/K}$	
1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^3 \text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)$
Coulomb's law constant, $k = 1/(4\pi\epsilon_0) = 9.0 \times 10^9 (\text{N}\cdot\text{m}^2)/\text{C}^2$	
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7} (\text{T}\cdot\text{m})/\text{A}$
Magnetic constant, $k' = \mu_0/(4\pi) = 1 \times 10^{-7} (\text{T}\cdot\text{m})/\text{A}$	
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^5 \text{ N/m}^2 = 1.0 \times 10^5 \text{ Pa}$

UNIT SYMBOLS	meter, m	mole, mol	watt, W	farad, F
	kilogram, kg	hertz, Hz	coulomb, C	tesla, T
	second, s	newton, N	volt, V	degree Celsius, °C
	ampere, A	pascal, Pa	ohm, Ω	electron volt, eV
	kelvin, K	joule, J	henry, H	

PREFIXES		
Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin \theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan \theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

- I. The frame of reference of any problem is inertial unless otherwise stated.
- II. The direction of current is the direction in which positive charges would drift.
- III. The electric potential is zero at an infinite distance from an isolated point charge.
- IV. All batteries and meters are ideal unless otherwise stated.
- V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{J} = \int \vec{F} dt = \Delta \vec{p}$$

$$\vec{p} = m\vec{v}$$

$$|\vec{F}_f| \leq \mu |\vec{F}_N|$$

$$\Delta E = W = \int \vec{F} \cdot d\vec{r}$$

$$K = \frac{1}{2} m v^2$$

$$P = \frac{dE}{dt}$$

$$P = \vec{F} \cdot \vec{v}$$

$$\Delta U_g = mg\Delta h$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

$$I = \int r^2 dm = \sum mr^2$$

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

$$v = r\omega$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

$$K = \frac{1}{2} I \omega^2$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

a = acceleration
 E = energy
 F = force
 f = frequency
 h = height
 I = rotational inertia
 J = impulse
 K = kinetic energy
 k = spring constant
 ℓ = length
 L = angular momentum
 m = mass
 P = power
 p = momentum
 r = radius or distance
 T = period
 t = time
 U = potential energy
 v = velocity or speed
 W = work done on a system
 x = position
 μ = coefficient of friction
 θ = angle
 τ = torque
 ω = angular speed
 α = angular acceleration
 ϕ = phase angle

$$\vec{F}_s = -k\Delta \vec{x}$$

$$U_s = \frac{1}{2} k (\Delta x)^2$$

$$x = x_{\max} \cos(\omega t + \phi)$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

$$T_p = 2\pi \sqrt{\frac{\ell}{g}}$$

$$|\vec{F}_G| = \frac{Gm_1 m_2}{r^2}$$

$$U_G = -\frac{Gm_1 m_2}{r}$$

ELECTRICITY AND MAGNETISM

$$|\vec{F}_E| = \frac{1}{4\pi\epsilon_0} \left| \frac{q_1 q_2}{r^2} \right|$$

$$\vec{E} = \frac{\vec{F}_E}{q}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$E_x = -\frac{dV}{dx}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

$$U_E = qV = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$\Delta V = \frac{Q}{C}$$

$$C = \frac{\kappa \epsilon_0 A}{d}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$I = \frac{dQ}{dt}$$

$$U_C = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

$$R = \frac{\rho \ell}{A}$$

$$\vec{E} = \rho \vec{J}$$

$$I = Nev_d A$$

$$I = \frac{\Delta V}{R}$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$P = I \Delta V$$

A = area
 B = magnetic field
 C = capacitance
 d = distance
 E = electric field
 $\mathcal{E}$ = emf
 F = force
 I = current
 J = current density
 L = inductance
 ℓ = length
 n = number of loops of wire per unit length
 N = number of charge carriers per unit volume
 P = power
 Q = charge
 q = point charge
 R = resistance
 r = radius or distance
 t = time
 U = potential or stored energy
 V = electric potential
 v = velocity or speed
 ρ = resistivity
 Φ = flux
 κ = dielectric constant

$$\vec{F}_M = q\vec{v} \times \vec{B}$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$$

$$\vec{F} = \int I d\vec{\ell} \times \vec{B}$$

$$B_s = \mu_0 n I$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -L \frac{dI}{dt}$$

$$U_L = \frac{1}{2} L I^2$$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$$A = bh$$

Triangle

$$A = \frac{1}{2}bh$$

Circle

$$A = \pi r^2$$

$$C = 2\pi r$$

$$s = r\theta$$

Rectangular Solid

$$V = \ell wh$$

Cylinder

$$V = \pi r^2 \ell$$

$$S = 2\pi r \ell + 2\pi r^2$$

Sphere

$$V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$

Right Triangle

$$a^2 + b^2 = c^2$$

$$\sin \theta = \frac{a}{c}$$

$$\cos \theta = \frac{b}{c}$$

$$\tan \theta = \frac{a}{b}$$

A = area

C = circumference

V = volume

S = surface area

b = base

h = height

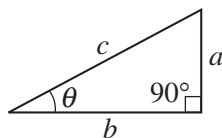
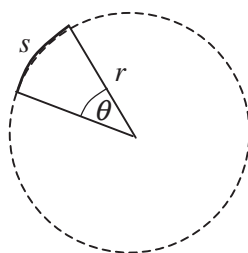
ℓ = length

w = width

r = radius

s = arc length

θ = angle



CALCULUS

$$\frac{df}{dx} = \frac{df}{du} \frac{du}{dx}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a \cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a \sin(ax)$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax)$$

VECTOR PRODUCTS

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$|\vec{A} \times \vec{B}| = AB \sin \theta$$

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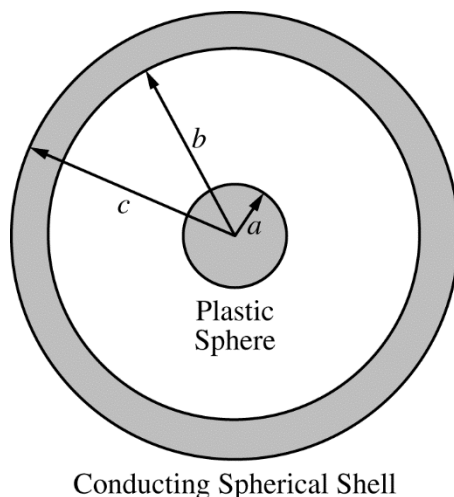
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β , r , a , and physical constants, as appropriate.
 - (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.
 - i. $r < a$
 - ii. $a < r < b$
 - (b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.
 - i. $r < a$
 - ii. $a < r < b$
 - (c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.
 - i. Determine the charge on the inner surface of the conducting shell.
Justify your answer.
 - ii. Determine the charge on the outer surface of the conducting shell.

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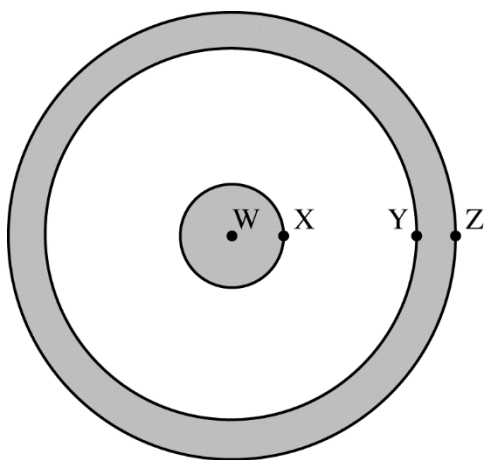
(d)

- i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.

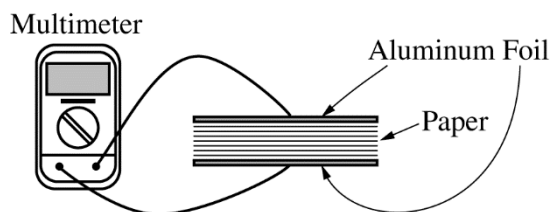


- ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

___ W ___ X ___ Y ___ Z



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2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060 \text{ m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

- (a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

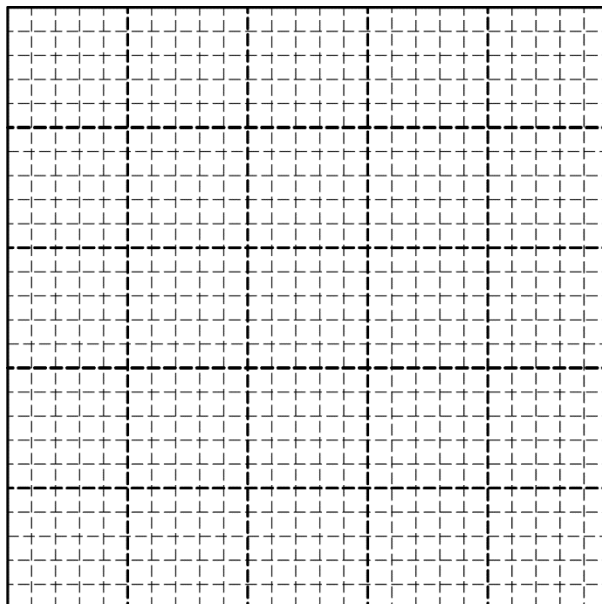
Vertical axis: _____

Horizontal axis: _____

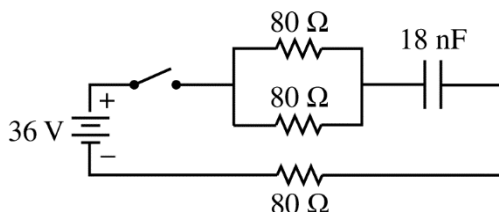
Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

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- (b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



- (c) Using the straight line, calculate a dielectric constant for the paper.



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be $18\ \text{nF}$. The student uses this uncharged capacitor to build a circuit using wire, a $36\ \text{V}$ battery, 3 identical $80\ \Omega$ resistors, and an open switch, as shown in the figure above.

- (d) Calculate the current in the battery immediately after the switch is closed.
- (e) Determine the time constant for this circuit.
- (f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.
- Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?
 _____ Larger than measured _____ Smaller than measured
 Explain experimentally what could account for this.
 - Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?
 _____ Larger than measured _____ Smaller than measured
 Explain experimentally what could account for this.

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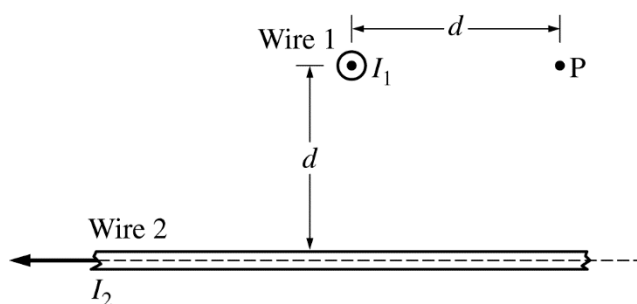


Figure 1. Side view

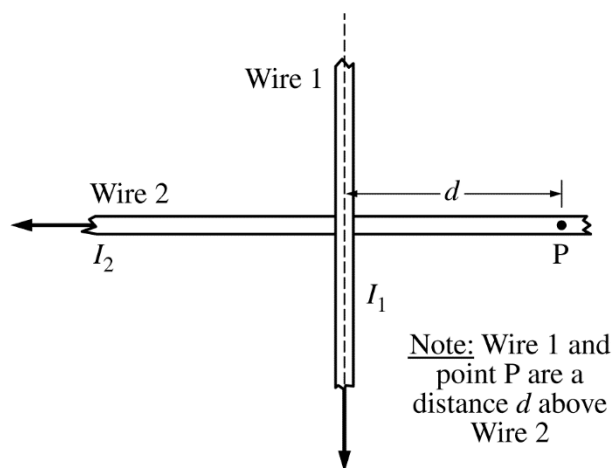


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I , d , and physical constants, as appropriate.
- Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.
 - Derive an expression for the magnitude of the net magnetic field at point P.
 - Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.
 - Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2? Assume gravitational effects are negligible.
 - ☐ Wire 1 will not move.
 - ☐ Wire 1 will move upward as viewed in Figure 1.
 - ☐ Wire 1 will move downward as viewed in Figure 1.
 - ☐ Wire 1 will rotate clockwise as viewed in Figure 2.
 - ☐ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

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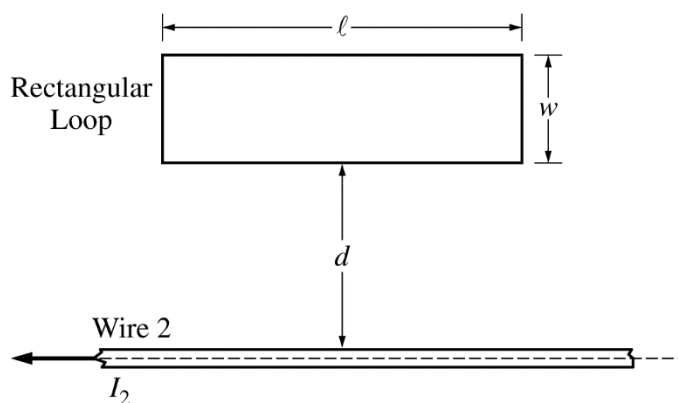


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ , width w , and resistance R . The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1} .

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t .

_____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

_____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

_____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

_____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation

$\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0 , d , r , R , w , k , ℓ , and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3?

_____ Clockwise _____ Counterclockwise

_____ Undefined, because there is no current induced in the loop

Justify your answer.

STOP

END OF EXAM